Mathématiques Sans Frontières

Discovery Questions

Even partial solutions and attempts can get marks ♦ Neat and careful work is important ♦ Hand in only one team answer sheet for each question.

Question 1 (7 marks): Bike and Run

Chloé und Lucille wollen an ihrem ersten *Bike and Run* teilnehmen. Beim *Bike and Run* absolvieren beide Mitglieder eines Teams dieselbe Strecke mit nur einem Fahrrad, das abwechselnd benutzt werden darf. Eine Person im Team fährt also Fahrrad, während die andere läuft. Wenn sie vom Rad steigt, lässt sie es am Wegrand für die andere Person stehen und setzt das Rennen zu Fuß fort. Wenn die Teamkollegin das Fahrrad zu Fuß erreicht hat, steigt sie auf und setzt das Rennen auf dem Rad fort. Es darf beliebig oft gewechselt werden. Chloé läuft 8 km pro Stunde. Mit dem Rad fährt sie 20 km pro Stunde. Lucille läuft 10 km pro Stunde. Mit dem Rad fährt sie 16 km pro Stunde. Die Wettkampfstrecke ist 27 km lang.

Wie müssen sich die beiden Mädchen das Rennen einteilen, um zusammen die Ziellinie zu überqueren?

In welcher Zeil absolvieren sie die 27 km?

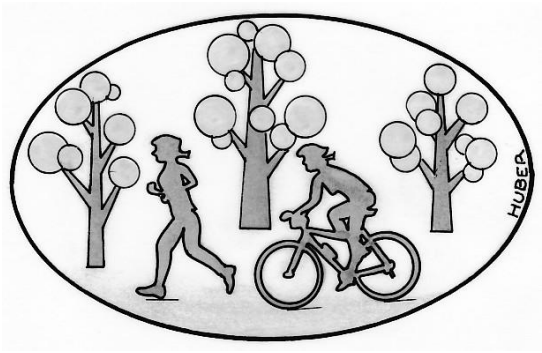
Chloé et Lucille veulent participer à leur premier « Bike and Run » de 27 km. Il s'agit d'une épreuve sportive dans laquelle une personne court pendant que l'autre roule à vélo. Un seul vélo est utilisé par les deux concurrentes qui se relaient.

Chaque fois qu'une des deux filles descend du vélo, elle le laisse sur le bord du parcours et continue la course à pied. Quand sa coéquipière arrive à pied, elle prend le vélo et se met à rouler.

Chloé court à 8 km/h et roule à vélo à 20 km/h. Lucille court à 10 km/h et roule à vélo à 16 km/h.

Comment les deux filles peuvent- elles s'organiser pour arriver ensemble sur la ligne d'arrivée ?

Combien de temps durera leur course ?



Chloé e Lucille partecipano in coppia alla loro prima corsa "Bike and Run" lunga 27 km. Si tratta di una competizione sportiva nella quale un partecipante corre mentre il compagno pedala in bicicletta. Ogni coppia può usare una sola bicicletta.

Ogni volta che una delle due concorrenti scende dalla bicicletta, la lascia sul bordo della strada e prosegue a piedi. Quando la sua compagna arriva a piedi, la recupera e prosegue pedalando.

Chloé corre a 8 km/h e pedala a 20 km/h mentre Lucille corre a 10 km/h e pedala a 16 km/h.

Spiegate come la coppia si deve organizzare per arrivare insieme al traguardo. Quanto dura la corsa?

Chloé y Lucile quieren participar en su primer « Bike and Run » de 27 km. Se trata de una competición deportiva en la que una persona corre mientras la otra pedalea. Se utiliza una única bici para las dos participantes que se relevan.

Cada vez que una de las chicas se baja de la bici, la deja en el borde del recorrido y sigue la carrera a pie. Cuando su compañera llega a pie, coge la bici y comienza a pedalear.

Chloé corre a 8 km/h y pedalea a 20 km/h. Lucille corre a 10 km/h y pedalea a 16 km/h.

¿Cómo pueden organizarse las dos chicas para llegar juntas a la meta?

¿Cuánto tiempo durará la carrera?

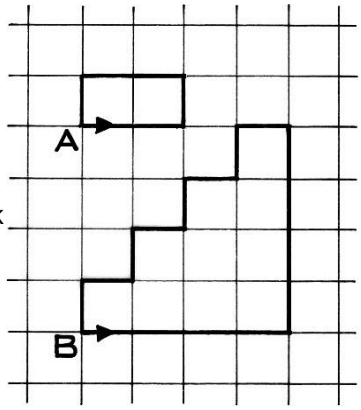
Question 2 (5 marks): *Ask the programme*

We have a robot that is programmed using only four instructions. These instructions are

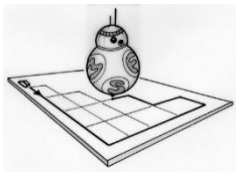
Move forward: the robot moves forward through one or more squares;
Turn right: the robot turns 90 degrees clockwise;
Turn left: the robot turns 90 degrees anticlockwise;
Repeat x times [...] : the robot repeats the instructions in the brackets x times.

The length of a programme is measured by counting the number of times the commands (words in bold) appear in it.

For example, consider the robot facing the right of the page at position A. Here is a programme that takes the robot back to position A with the fewest commands possible:



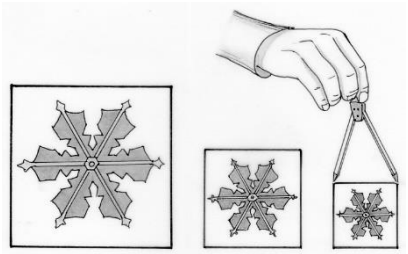
Repeat 2 times
[**Move** 2 squares; **Turn** left; **Move** 1 square; **Turn** left]



Write a programme with as few commands as possible to follow the path indicated when the robot starts facing right at point B.

Question 3 (7 marks) : *Sum of areas*

Given the three squares shown below:



Without measuring the lengths of the sides of these squares, construct a square whose area equals the sum of the areas of the three squares.

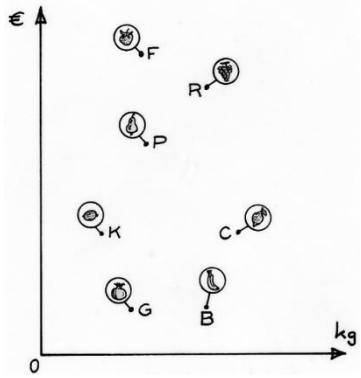
You may use a pair of compasses to measure sides (rulers are not allowed). The snowflakes within each square are not required in your construction.

Question 4 (5 marks): *Hold the line*

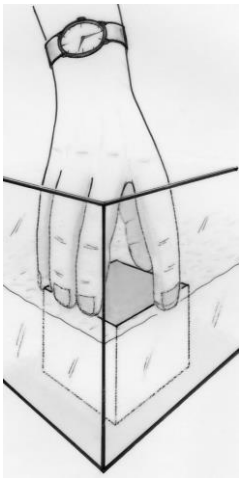
A merchant sells different kinds of fruit. He fills seven bags, each with a single kind of fruit, and decides the price for each bag. Instead of labelling the bags, he creates a graph where each point represents a bag of fruit. The mass of the bag (in kilograms) is shown on the horizontal axis, while the vertical axis represents its corresponding price in euros.

With the help of the graph, find two bags of fruit that have the same price per kilogram.

Further, write down the labels of the seven bags in increasing order of price per kilogram. Explain the order you have written down.



Question 5 (7 marks): *Flush fluid*



An aquarium with right-angled corners contains water. The dimensions of the base, measured in centimetres, are whole number values.

Jeannette places a cube of side 10 cm at the bottom of the aquarium. The water level is now equal to the height of the cube.

She replaces this cube with a cube of side 20 cm. The new water level is equal to the height of this new cube.

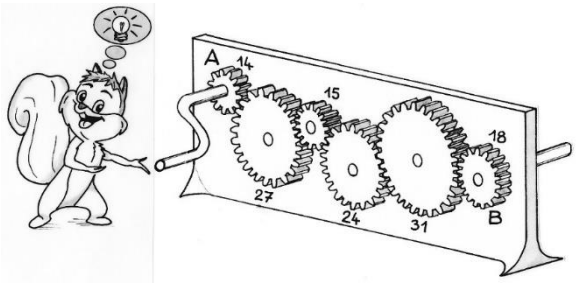
Give the dimensions of the base of the aquarium and the volume of the water. Justify your answer.

Question 6 (5 marks): *Cogs in a wheel*

In the device shown in the picture, the number beside each cog gives the number of teeth it has.

Do cogs A and B turn in the same direction?

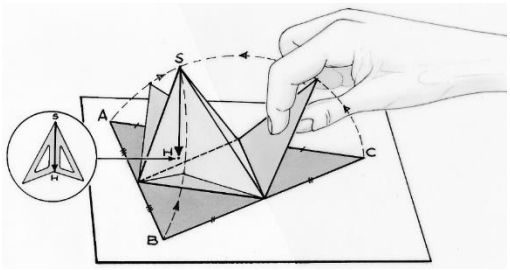
What is the minimum whole number of turns that cog A must make in order that both cogs A and B each make a non-zero number of complete turns?



How many complete turns does cog B then make?
Justify your answer.

Question 7 (7 marks): *Tetrahedron of mid points*

A simple triangle ABC can easily be used to easily construct a tetrahedron, provided that each of its three angles is an acute angle. To do this, simply draw lines joining midpoints of the sides. Then rotate the three triangles with these lines as base to create an apex S as shown.



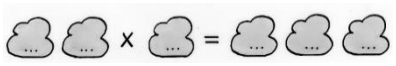
The resulting tetrahedron is "equifacial" in the sense that its four faces are the same size.

We are interested in the foot H of the height of the tetrahedron from S. By raising each face, we can see that H lies inside the original triangle ABC.

Construct the point H, the base of the height of the tetrahedron inside the original triangle ABC.

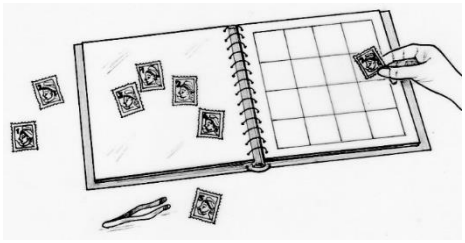
Question 8 (5 marks): *Six figures*

Complete the equation using all the digits 1 to 6 so that the operation is exact.



Question 9 (7 marks): *Stamp duty*

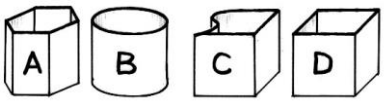
In his stamp collection, Gerard has stamps worth 1€, 2€, 3€, 4€ or 5€. Using sixteen of these stamps, he can fill all the boxes in a 4 by 4 grid so that no row, no column and no diagonal or line parallel to a diagonal has two stamps of the same value.



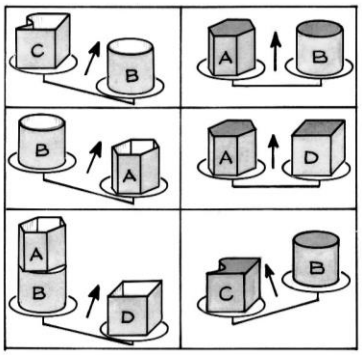
Provide one such possible distribution of stamps on the grid.

Question 10 (10 marks): *Balancing act*

We have four containers A, B, C and D.



Some of the containers are placed, either empty or completely filled with water, on a scale. The picture on the right shows the outcomes of the weighing. In the pictures, the containers on the left are empty; those on the right are full.



With the help of the picture, order the containers in each of the following two ways:

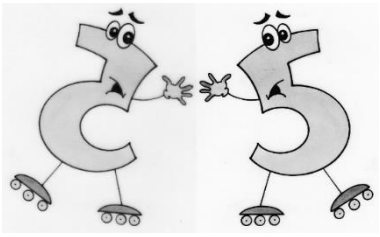
- ***From lightest to heaviest when empty;***
- ***In ascending order of their volumes.***

Senior classes only

Question 11 (5 marks): *Pair of pentahedra*

A pentahedron is a polyhedron with five sides.

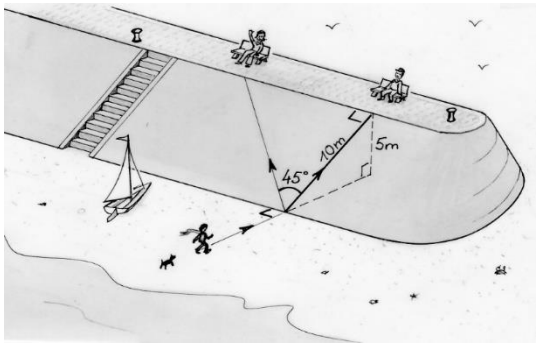
On your answer sheet, draw using cavalier projection, two pentahedra which do not have the same number of edges.



Question 12 (7 marks): *Malo's wall*

Lily, who is coming from the beach, wants to climb up Malo-les-Bains's sea wall. This sea wall is 5m high. The shortest path up the wall, and therefore the steepest, is 10m in length, with a slope of 5 in 10 or 50%.

Tired, Lily decides to walk in a straight line at a 45° angle away from the shortest path.



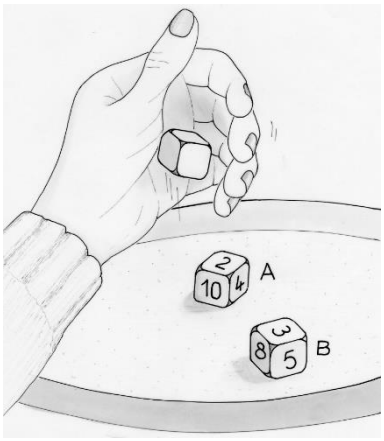
Calculate, as a percentage, the slope of this new path. At what angle should Lily walk away so that the slope is 25%? Justify your answer.

Question 13 (10 marks): *Dice Challenge*

On a rainy afternoon, Anatole and Barnabé had fun making unusual dice: the numbers on opposite faces are equal. Anatole makes die A using the numbers 2, 4 and 10, and Barnabé makes die B using the numbers 3, 5 and 8. They throw their dice at the same time. Each face has the same probability of appearing. A player wins when the number shown on the top face of his die is greater than the top face of the opponent's die.

What is the probability that Anatole wins? Explain.

Chloe, their sister, arrives and issues the following challenge: "Make me a die the same type with three different numbers so that if I play against Anatole I have less than a 50% chance of winning, and if I play against Barnabé I have more than a 50% chance of winning.

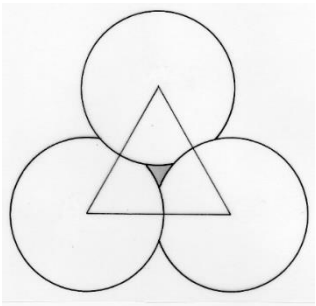


Give an example of a die that meets this challenge.

Question 13 Pro (10 marks): *Geometry under glass*

Three disc-shaped coasters, each of diameter 10cm, are placed on a restaurant table.

While playing with the coasters, I arrange them so that their centres form the vertices of and equilateral triangle. I move the coasters closer until the grey area shown in the diagram on the right disappears. The three centres still form the vertices of an equilateral triangle.



Determine the length of a side of the equilateral triangle when the grey area disappears.

Dynamic geometry software may be used to answer this question.